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# $E_6$ multiplets and unification in extra dimensions

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## Abstract

We study the effect of all matter multiplets contained in **27** representation of  $E_6$  GUT on gauge coupling unification in extra dimensions. Extra members of **27** multiplets of all three generations have their ‘zero modes’ near  $m_t$  such that they can be directly probed. From TeV scale onwards extra dimensions open up, theory becomes N=2 supersymmetric and gauge couplings unify or they do not depending on how we distribute matter fields and gauge fields in bulk and brane. We find three such possible embedding which will lead to perfect gauge coupling unification below 100 TeV region for one extra dimension and lower than that if number of extra dimensions is larger.

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Supersymmetry breaking is a hard problem. However attempts have been made to link lightness of supersymmetry breaking scale to some large radius of compactifications in string theory[1]. Independently large extra dimensions of millimeter size have been invoked to stabilize gauge hierarchy problem as proposed in Ref.[2]. This way to tackle hierarchy problem is independent of the existence of low energy supersymmetry. Moreover special features of open string theory[3] have also been used to try and bring down the fundamental string scale itself to TeV region[4, 5]. All these new results give good motivation for studying theories where extra dimensions show up much below  $10^{18}$  GeVs. One possible consequence of such large extra dimensions is that gauge coupling unification can happen in  $M_X = \text{few tens of TeVs}$ [6]. Then even though our unified theory in extra dimensions is non-renormalizable gauge coupling unification happens close enough to the low lying string scale in such a way that three gauge forces unify with gravity and side by side divergences of non-renormalizable effective gauge theory are properly handled by full string theory which takes over almost immediately.

Here we will study gauge coupling unification in  $E_6$  Grand Unified Theory (GUT)[7] in extra dimensions. We know that fermions are unified in the **27** representation of  $E_6$  which can be decomposed as

$$\begin{aligned}
E_6 \supset & \quad SU(3) \times SU(2) \times U(1) \\
27 \supset & \quad \underbrace{(3, 2, 1/6) + (\bar{3}, 1, -2/3) + (1, 1, 1)}_{10 \text{ of } SU(5)} + \underbrace{(\bar{3}, 1, 1/3) + (1, 2, -1/2)}_{\bar{5} \text{ of } SU(5)} + \underbrace{(1, 1, 0)}_{RH \text{ neutrino}} \\
& \quad + \underbrace{(\bar{3}, 1, 1/3) + (3, 1, -1/3) + (1, 2, -1/2) + (1, 2, 1/2) + (1, 1, 0)}_{exotic \text{ fermions}}.
\end{aligned}$$

Because two extra singlets will not affect gauge coupling unification we will work with low energy multiplets that can be thought as  $5 + \bar{5}$  of  $SU(5)$ . We know that in 4 dimensions introduction of complete  $SU(5)$  multiplets do not affect gauge coupling unification but the unified coupling can become non-perturbative before GUT scale is reached. However as we will learn below this is not necessarily the case in the presence of extra dimensions. Because this is a three generation analysis we can have full three copies of  $5 + \bar{5}$  hanging well below the GUT scale due to  $E_6$  symmetry. That is  $5 + \bar{5}$  do not pair up and become as heavy as the GUT scale. Then we have four possible types of extra matter multiplets. We assume that their masses are near the top quark mass  $m_t$ . In extra dimensional models we use the terminology that their ‘zero modes’ are near  $m_t$ . At the scale  $\mu_0 = 1 \text{ TeV}$   $\delta$  number of extra dimensions open up and excited Kaluza-Klein states starts to show. Under standard model

gauge group we label extra matter superfields as,

$$L_1 = (1, 2, -1/2) \ ; \ D_1 = (\bar{3}, 1, 1/3) \ ; \ L_2 = (1, 2, 1/2) \ ; \ D_2 = (3, 1, -1/3). \quad (1)$$

Contribution of  $L_1$  and  $L_2$  to beta coefficients will be same as usual lepton doublet whereas contribution of  $D_1$  and  $D_2$  will be same as down type antiquark. Gauge couplings evolve[6]

<i>fields</i>	<i>representation</i>	$\tilde{\beta}_3$	$\tilde{\beta}_2$	$\tilde{\beta}_1$
$H_1$	$(1, 2, \frac{1}{2})$	0	1	1
$H_2$	$(1, 2, -\frac{1}{2})$	0	1	1
$Q$	$(3, 2, -\frac{1}{6})$	$\eta_Q \ 2$	$\eta_Q \ 3$	$\eta_Q \ \frac{1}{3}$
$\overline{D}$	$(\bar{3}, 1, \frac{1}{3})$	$\eta_U \ 1$	0	$\eta_U \ \frac{2}{3}$
$\overline{U}$	$(\bar{3}, 1, -\frac{2}{3})$	$\eta_D \ 1$	0	$\eta_D \ \frac{8}{3}$
$L$	$(1, 2, -\frac{1}{2})$	0	$\eta_L \ 1$	$\eta_L \ 1$
$\overline{E}$	$(1, 1, 1)$	0	0	$\eta_E \ 2$
<i>gauge</i>	$(8, 3, 0)$	-6	-4	0

Table 1: contributions to  $\tilde{\beta}$  coefficients in (N=2) supersymmetric standard model. Extra  $E_6$  multiplets  $L_1$  and  $L_2$  contribute same as L and  $D_1$  and  $D_2$  contribute same as  $\overline{D}$

with energy via the following equation where we have redefined  $t = \frac{1}{2\pi} \ln(\Lambda)$ ,  $t_0 = \frac{1}{2\pi} \ln(\mu_0)$  and  $\alpha = g^2/4\pi$ . Here  $\Lambda$  is the cut-off scale where couplings are being evaluated.

$$\frac{d\alpha_i}{dt} = [(\beta_i - \tilde{\beta}_i) + \tilde{\beta}_i X_\delta e^{2\pi\delta(t-t_0)}] \alpha_i^2 \quad \text{and} \quad X_\delta = \frac{\pi^{\delta/2}}{\Gamma(1 + \delta/2)}. \quad (2)$$

$\Gamma$  is Euler Gamma function. Note that we must recover familiar Renormalization Group Equations (RGE) in the limit of either  $\tilde{\beta} = 0$  or  $\delta = 0$ . We have used the usual notation[9] that  $\beta$ s are 4-dimensional coefficients whereas  $\tilde{\beta}$ s are higher dimensional coefficients. Expressions of  $\tilde{\beta}$ s are given in detail in Tables (1) for N=2 supersymmetric standard model.

Let us quickly review the minimal scenario given by Dines Dudas and Gherghetta[6] which will set all our notations. There only gauge bosons and Higgs have bulk excitations whereas fermions remain at fixed points. Two Higgs doublets  $H_1$  and  $H_2$  are embedded in a single  $N = 2$  Higgs superfield. All  $\delta$  extra dimensions open up simultaneously at scale  $\mu_0$  which is a free parameter. Below the scale  $\mu_0$  we have N=1 supersymmetry whereas above  $\mu_0$  we have N=2 supersymmetry. In this case  $\beta = (\frac{33}{5}, 1, -3)$  and  $\tilde{\beta} = (\frac{3}{5}, -3, -6)$ . Following

Eqn.(2) we write

$$\begin{aligned}\frac{d\alpha_Y}{dt} &= [6 + 3/5 X_\delta e^{2\pi\delta(t-t_0)}] \alpha_Y^2, \\ \frac{d\alpha_2}{dt} &= [4 - 3 X_\delta e^{2\pi\delta(t-t_0)}] \alpha_2^2, \\ \frac{d\alpha_3}{dt} &= [3 - 6 X_\delta e^{2\pi\delta(t-t_0)}] \alpha_3^2.\end{aligned}\tag{3}$$

Using two loop renormalization group equations for the gauge couplings below  $\mu_0$  we can solve Eqn. (3) numerically, the results are given in Fig. (1) CASE 1. Now let us examine the unification condition discussed by DDG by defining ratio

$$R_{ij} = \frac{\tilde{\beta}_i - \tilde{\beta}_j}{\beta_i - \beta_j}.\tag{4}$$

Then unification is achieved when we have,

$$R_{12} = R_{13} = R_{23}.\tag{5}$$

For DDG case we have  $\frac{R_{12}}{R_{13}} = 0.94$  and  $\frac{R_{13}}{R_{23}} = 0.92$ . Thus gauge coupling unification is only approximate. Let us examine scenario 3 discussed by Carone[8]. There are two  $5 + \bar{5}$  of which leptons have bulk excitations (total 5 N=1 pairs) and one generation of electron have bulk excitation (1 N=1 pair). All fermions have zero modes at scale  $O(m_W)$ . SU(3) gauge bosons stay on the boundary. Then we have  $R_{12} = R_{13} = R_{23} = 1/2$  which gives perfect unification. We see this case is very similar to our  $E_6$  scenario except that we want to keep zero modes of all three generations of **27** near  $m_t$ . This fixes  $\beta$  coefficients below  $\mu_0$  to be  $\beta = (48/5, 4, 0)$ . However adding one more  $5 + \bar{5}$  near  $m_t$  will keep the difference  $\beta_i - \beta_j$  unchanged which occurs in the denominator in Eqn. (4). Thus couplings will still unify. Keeping only SU(2) and U(1) gauge bosons in the bulk we calculate unification condition using Eqn. (5),

$$\eta_E = \frac{3\eta_L - 13}{2}.\tag{6}$$

For  $\eta_L = 5, \eta_E = 1$  we get CASE 2

$$\beta = (48/5, 4, 0) \quad , \quad \tilde{\beta} = (24/5, 2, 0)\tag{7}$$

Then three generations of full **27** multiplets contribute to evolution of gauge couplings from the scale  $m_t$  onwards and changes coefficient  $\beta$  even though their effect does not show up in the difference  $\beta_i - \beta_j$  which appears in the quantity  $R_{ij}$ .

Next let us consider the case when above  $\mu_0$  SU(3) and U(1) gauge bosons have bulk excitations but SU(2) do not. Again because bulk matter transform under bulk gauge group only  $\overline{U}, \overline{D}$  and  $\overline{E}$  can have bulk excitations. Then we calculate unification condition using Eqn. (5)

$$\eta_D = \frac{14 - 2\eta_E - 5\eta_U}{3}. \quad (8)$$

and two pairs of  $\overline{E}$  and two pairs of  $\overline{U}$  can have bulk excitations. Then below and above  $\mu_0$  the beta coefficients are

$$\beta = (48/5, 4, 0) \quad , \quad \tilde{\beta} = (28/5, 0, -4). \quad (9)$$

In this scenario we get,

$$R_{12} = R_{13} = R_{23} = 1. \quad (10)$$

The unification picture is given in Fig (1) CASE 3. Second case is when three  $\overline{D}$  pairs and one  $\overline{U}$  pair have bulk excitations. In this case we get,

$$\beta = (48/5, 4, 0) \quad , \quad \tilde{\beta} = (14/5, 0, -2). \quad (11)$$

In this scenario we get,

$$R_{12} = R_{13} = R_{23} = 1/2. \quad (12)$$

The unification picture is given in Fig (1) CASE 4.

Even though strictly speaking Eqn(5) is valid at one loop, below  $\mu_0$  we have used two loop running assuming all superpartners and extra  $E_6$  matter near the scale  $m_t$ . This does not affect gauge coupling unification appreciably as we see from Fig (1). Also from Fig (1) we see that for  $\delta = 2$  unification scale is much lower than  $\delta = 1$  case. We would like to stress that we have not introduced any new multiplet in adhoc basis. We have only concentrated on multiplets contained within **27** representation of  $E_6$  group. In next generation colliders ‘zero modes’ of these the  $E_6$  exotic particles may be discovered[11]. Furthermore excited Kaluza-Klein modes of gauge bosons and matter fields will also be rigorously searched [12] in near future.

We have ignored heavy threshold corrections in the paper. This is because we have assumed that heavy GUT multiplets exist at or above the unification scale and they are degenerate in mass. This is in the spirit of an extended survival hypothesis in conventional unified models. However in case some odd members of a heavy GUT multiplets have masses below the unification scale we will need to include heavy threshold corrections[10] to our results.

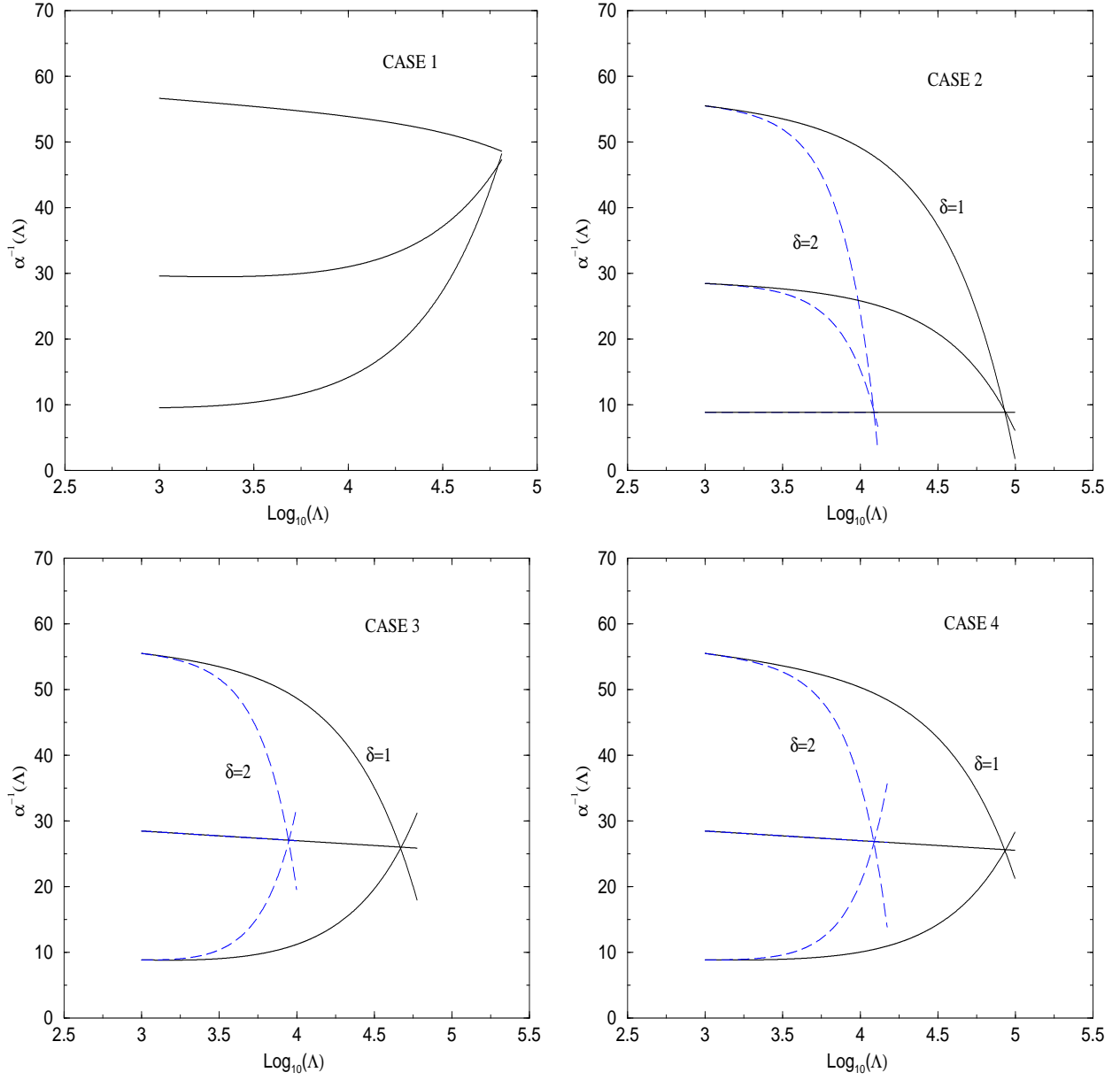


Figure 1: CASE 1: Original DDG case with  $\mu_0 \sim 1$  TeV where only gauge bosons and Higgs scalar has bulk excitations. Gauge couplings do not meet precisely. CASE 2: Only SU(2) and U(1) has bulk excitations. Three **27** generations at scale  $m_t$  of which leptons have bulk excitations with  $\eta_L = 5, \eta_E = 1$  above  $\mu_0$ . CASE 3: Only SU(3) and U(1) has bulk excitations. All three 27 multiplets have zero modes at scale  $m_t$ , above  $\mu_0$   $\eta_E = 2, \eta_U = 2$  have bulk excitations. CASE 4: Only SU(3) and U(1) has bulk excitations. All three 27 multiplets have zero modes at scale  $m_t$ , above  $\mu_0$   $\eta_D = 3, \eta_U = 2$  have bulk excitations.

In a recently updated study Kang and Langacker have studied the discovery limits[13] of exotic  $E_6$  multiplets in Fermilab Tevatron and CERN LHC. They conclude that multiplets as light as 200 GeV can be probed directly in Tevatron and as light as 1 TeV in LHC. A natural question in this context is that are we allowed to put exotic  $E_6$  multiplets will masses near  $m_t$ ? In another words are they excluded by direct and indirect experimental searches? The answer is that with present experimental accuracy exotic  $E_6$  multiplets are allowed at around the scale  $m_t$ [14].

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